CBCS SCHEME

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18MCM11

First Semester M.Tech. Degree Examination, Jan./Feb. 2021
Applied Mathematics

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. An object of mass 10 kg is released from the rest 1000 m above the ground and allowed to fall under the gravity. Assume $g = 9.81\ m/sec^2$ and force due to resistance is proportional to velocity with proportional constant C = 10 N-sec/m. Determine the velocity and when the object will strike the ground. **(12 Marks)**

b. Use the method of iteration to find a positive root of the equation $xe^x = 1$. Give your answer correct to three decimal places. **(08 Marks)**

OR

2 a. Derive the analytical solution of freely falling body parachutist in the form

$$V = \frac{gm}{c}\left[1 - e^{-c/m\,t}\right]$$ **(08 Marks)**

b. Derive the series $\log_e\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right)$ and use it to compute the value of $\log_e(1.2)$, correct to seven decimal places. If, instead the series for $\log_e(1 + x)$ is used, how many terms must be taken to obtain the same accuracy for $\log_e(1.2)$? **(12 Marks)**

Module-2

3 a. Apply Muller's method to find the smaller positive root of the equation $x^3 - 5x + 1 = 0$ in (0, 1) (perform three iterations). **(10 Marks)**

b. Use Romberg's method to compute $\int_0^1 \frac{1}{1+x}dx$. Correct to three decimal places. **(10 Marks)**

OR

4 a. Evaluate $\int_{0.2}^{1.4}(\cos x + \ell nx - e^x)dx$ by (i) Trapezoidal rule (ii) Simpson's 1/3[rd] rule (iii) Simpson's 3/8[th] rule (iv) Weddle's rule, by taking 7 ordinates. **(12 Marks)**

b. From the table given below, compute y'(0.2) and y''(0.2.

x	1	1.2	1.4	1.6	1.8	2	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

(08 Marks)

Module-3

5 a. Using the partition method, solve the system of equations:

$2x + 4y + 3z = 4;\quad y + z = 1;\quad 2x + 2y - z = -2$ **(10 Marks)**

b. Solve the system equation

$3x_1 - x_2 + 2x_3 = 12;\quad x_1 + 2x_2 + 3x_3 = 11;\quad 2x_1 - 2x_2 - x_3 = 2$

By Crout's reduction technique. **(10 Marks)**

OR

6 a. Solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$ using Gauss elimination method. **(10 Marks)**

b. Solve the system of equation $x+2y+3z=5$, $2x+8y+22z=6$, $3x+22y+82z=-10$ by Cholesky method. **(10 Marks)**

Module-4

7 a. Find all eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$ by Jacobi's method. **(10 Marks)**

b. Determine eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$ by Given's method. **(10 Marks)**

OR

8 a. Using the Householder's transformation reduce the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ into tridiagonal matrix. **(10 Marks)**

b. Find all the eigen values of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ using the Rutishauser method. **(10 Marks)**

Module-5

9 a. Show that $\{V_1, V_2, V_3\}$ is an orthonormal basis of $\mathbb{R}^3$, where

$$V_1 = \begin{bmatrix} 3/\sqrt{11} \\ 1/\sqrt{11} \\ 1/\sqrt{11} \end{bmatrix}, \quad V_2 = \begin{bmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}, V_3 = \begin{bmatrix} -1/\sqrt{66} \\ -4/\sqrt{66} \\ 7/\sqrt{66} \end{bmatrix}$$ **(10 Marks)**

b. If $X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $X_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, then $\{X_1, X_2, X_3\}$ is linearly independent. Construct an orthogonal basis for w. **(10 Marks)**

OR

10 a. Find a least squares solution of the inconsistent system $AX = b$ for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$. **(10 Marks)**

b. A transformation T is linear, then prove that $T(0) = 0$. **(05 Marks)**

c. Let $T: R^n \to R^m$ be a linear transformation then prove that $T(x) = Ax$ where A is a matrix of order $m \times n$. **(05 Marks)**